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A MATHEMATICAL DECISION MODEL FOR BANKS FINANCE INVESTMENT

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Abstract- An investment is the current commitment of money or material resources with the expectation of reaping future benefits. Also investment is a long term planning at least one year, with low or moderate risks having low or moderate of return. In case planning is short term, (few days or months) risk is high with high rates of return. In this paper we have studied the Union Bank of India's finance investment policy and decision making procedures by analyzing the NPA and history of the granted loans to the customers. The problem is formulated in form of a constrained linear programming problem and solutions are obtained by using simplex algorithm technique with the use of MATLAB codes.
Keywords – NPA, Simplex algorithm, Binary integer program, Linear programming

1. INTRODUCTION

An investment is the current commitment of money or material resources with the expectation of reaping future benefits. Also investment is a long term planning at least one year, with low or moderate risks having low or moderate of return. In case planning is short term, (few days or months) risk is high with high rates of return. Investment decisions are influenced by hearsay, market psychology and resort to borrowed funds. Market psychology depends on investment analytic descriptions or abstract terms such as pur- pose, time risks, tools, financial data monitor and adjustment. Each financial product or investment program has rules restrictions, time commitment and cost associate with it. Establishing a time frame for each purpose or goal al- lows us to make better choices about the tools we use to achieve the purpose. In this paper the introduction descripts a brief literature defining investment decision support technical terms such as expected return or risks, portfolio and steps in decision process of investment analysis. In the second section we discuss types of investment and investment calculation attributes. The third section contain the formulation of a constrained Linear Programming modeled investment problem and calculate the Optimal decision variable values using simplex algorithm solver MATLAB and TORA tools. In the fourth section we discuss the application of binary integer program to decide optimum profitable investment projects from a set of listed investment project outcome values.

A proper investment decision process helps the service provider and service availing people happier. The banks and nations finance servicing units calculate effective in- vestment policies before execution a financial or infrastructure development projects. Governments people welfare schemes are also needed proper financial studies prior to implement the scheme. Each financial product or investment program has certain rules, restrictions, time commitments, and cost associates with it. Establishing a time frame for each purpose and goal fulfilment, it allow us to make better choices about the tools. The insurance company policies and participating people have certain type of investments namely, CD, Bonds, Stock, Real estate and commodities. The investments are transacting through banks. The programmable investment calculator help in determining effective set of variables concerning investment with a fixed rate of return, There are four crucial elements such as Return rate, starting amount, investment length and additional contribution usually calculated through programmable investment calculators.

2. LITERATURE DEFINING INVESTMENT DECISION SUPPORT TECHNICAL TERMS

Expected return and risks: Realized returns meaning actual return might be more or less than the expected return. In case actual return is less than the expected return is called the risk.

Steps in decision process: these are the security analysis and portfolio management. The value of the security is a function of future earnings from the security and the risk attached. After securities have been analyzed and valued, portfolio is selected and security is made.

Portfolio management: These are either passive investment strategy or active investment strategy. In passive strategy the investors make few changes in the portfolio, so that transaction costs, time and search costs are minimum. In active strategies investors believe that they can earn better returns by actively making changes in the portfolio.

In the next section, we formulate a bank's annual investment strategy using past years interest rate, NPA and corresponding constraints in the form of a linear programming algebraic model problem. We use the numerical values to write constraint inequalities for solving the problem using simplex algorithm through MATLAB and TORA.

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3. A CONSTRAINED LINEAR PROGRAMMING MODELED INVESTMENT PROBLEM AND THE OPTIMAL DECISION VARIABLE CALCULATION

A bank is in the process of devising a loan policy with the total funds should not exceed 4652 crore the following table provides the numerical data about the available types of loans. The numerical data consists of interest rate and NPA meaning Non Performing Assets. The NPAs are unrecoverable and produces no interest revenue.

S.No.	Type of loans	Interest rate	NPA
1.	Home	8.65	1.3462
2.	Vehicle	11.55	0.212
3.	Education	10.55	0.1749
4.	LAP	11.45	0.2624
5.	Personal	12.55	0.0712
6.	Other	12.90	0.583
7.	SMF	1.9	4.1745
8.	Agriculture	2.65	2.7255
9.	Industry	1.25	5.3450
10.	Service	1.25	12.2950

The bank managerial follows following strategies and restrictions to provide bank loan.

1. Total funds should not exceed 4652 Crore.
2. NPA should not exceed 16%.
3. Home, Vehicle Education, LAP, Personal and Other loans should not exceed 400 Crore.
4. SMF and Agriculture loans should not exceed 600 crore.
5. Industry and service loans should not exceed 3500 crore.
6. Home, Vehicle, Education, LAP, personal and Other loans should equal at least 8% of all loans.
7. SMF and Agriculture loans should equal at least 11% of all loans.
8. Industry and Service loans should equal to at least 60% of all loans.
9. Home Vehicle, Education, LAP, personal and Other loans should equal at least 73% of SMF and Agriculture loans.
10. SMF and Agriculture loans should equal at least 14% of Industry and Service loans.

The bank managerial want to determine the quantity of loans in each category so that NPA should be minimum and revenue is maximum.

Mathematical Models for variable and constraint determination:

Let us assign the variable for loan quantities in crores

x_1 = Home loans
x_2 = Vehicle loan
x_3 = Education loan
x_4 = LAP loan
x_5 = Personal loan
x_6 = Other loans
x_7 = SMF loans
x_8 = Agriculture loans
x_9 = Industry loan
x_{10} = Service loan.

The objective of the bank managerial is to maximize the difference of interest rev- enue and NPA. Since 1.3462% of Home loan is Non Performing Asset, the bank will receive interest on only 98.6538%of the home loan. Thus revenue from home loan will be .0865 × .986538x1. In this way the total interest is

f (x1, x2, ..., x10) = .0865 × .986538x1 + .1155 × .99788x2 + .1055 × .998251x3 + .1145 × .997376x4 + .1255 × 0.99288x5 + .1290 × 0.99417x6 + .019 × .958255x7 +.0265 × .972745x8 + .0125 × .94655x9 + .0125 × .87705x10

f (x1, x2, ..., x10) = .085335537x1 + .11525514x2 + .1053154805x3 + .114199552x4 +.125410644x5 + .12824793x6 + .018206845x7 + .0257777425x8 + .011831875x9 + .010963125x10

Total NPA = 0.013462x1 + 0.00212x2 + 0.001749x3 + 0.002624x4 + 0.000712x5 + 0.00583x6 + 0.041745x7 + 0.027255x8 + 0.05345x9 + 0.12295x10

Maximize Z = Total interest -Total NPA

Maximize Z = .071873537x1 + .11313514x2 + .1035664805x3 + .111575552x4 + .124698644x5 + .12241793x6 − .023538155x7 − .0014772575x8 − .041618125x9 − .111986875x10

The constraints are determined from the investment strategies mentioned in the problem description.

Total funds should not exceed 4652 crore.

$x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 \leq 4652$

NPA should not exceed 16 %.

$0.013462x1 + 0.00212x2 + 0.001749x3 + 0.002624x4 + 0.000712x5 + 0.00583x6 + 0.041745x7 + 0.027255x8 + 0.05345x9 + 0.12295x10 \leq .16(x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10)$

Or

$-0.146538x1 - 0.15788x2 - 0.158251x3 - 0.157376x4 - 0.159288x5 - 0.15417x6 - 0.118255x7 - 0.132745x8 - 0.10655x9 - 0.03705x10 \leq 0$

Home, Vehicle, Education, LAP, Personal and Other loans should not exceed 400 crore.

$x1 + x2 + x3 + x4 + x5 + x6 \leq 400$

SMF and Other Agriculture loans should not exceed 600 crore.

$x7 + x8 \leq 600$

Industry and Service loans should not exceed 3500 crore.

$x9 + x10 \leq 3500$

Home, Vehicle, Education, LAP, Personal and Other loans should equal at least 8 % of all loans.

$x1 + x2 + x3 + x4 + x5 + x6 \geq .08(x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10)$ Or

$-0.92x1 - 0.92x2 - 0.92x3 - 0.92x4 - 0.92x5 - 0.92x6 + .08x7 + .08x8 + .08x9 + .08x10 \leq 0$

SMF and Other Agriculture loans should equal at least 11 % of all loans.

$.11x1 + .11x2 + .11x3 + .11x4 + .11x5 + .11x6 - 0.89x7 - 0.89x8 + .11x9 + .11x10 \leq 0$

Industry and Service loan should equal at least 60 % of all loans.

$.60x1 + .60x2 + .60x3 + .60x4 + .60x5 + .60x6 + .60x7 + .60x8 - 0.4x90.4x10 \leq 0$

Home, Vehicle, Education, LAP, Personal and other loans should equal at least 73 % of SMF and Other Agriculture loans.

$x1 + x2 + x3 + x4 + x5 + x6 \geq .73(x7 + x8)$

Or

$-x1 - x2 - x3 - x4 - x5 - x6 + 0.73x7 + 0.73x8 \leq 0$

SMF and Other Agriculture loans should equal at leans 14 % of Industry and Service loans.

$x7 + x8 \geq .14(x9 + x10)$

Or

$-x7 - x8 + 0.14x9 + 0.14x10 \leq 0$

Non negativity.

$x1 \geq 0, x2 \geq 0, x3 \geq 0, x4 \geq 0, x5 \geq 0, x6 \geq 0, x7 \geq 0, x8 \geq 0, x9 \geq 0, x10 \geq 0$

Output using MATLAB Version R2017a:

LPP

f =

0.0719 0.1131 0.1036 0.1116 0.1247 0.1224 -0.0235 -0.0015 -0.0416 -0.1120

A =

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
-0.1465	-0.1579	-0.1583	-0.1574	-0.1593	-0.1542	-0.1183	-0.1327	-0.1066	-0.0371
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0
0	0	0	0	0	0	1.0000	1.0000	0	0
0	0	0	0	0	0	0	0	1.0000	1.0000
-0.9200	-0.9200	-0.9200	-0.9200	-0.9200	-0.9200	0.0800	0.0800	0.0800	0.0800
0.1100	0.1100	0.1100	0.1100	0.1100	0.1100	-0.8900	-0.8900	0.1100	0.1100
0.6000	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000	0.6000	-0.4000	-0.4000
-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	0.7300	0.7300	0	0
0	0	0	0	0	0	-1.0000	-1.0000	0.1400	0.1400

b =

4652

0 400 600 3500 0 0 0 0 0

Aeq =

[]

Beq =

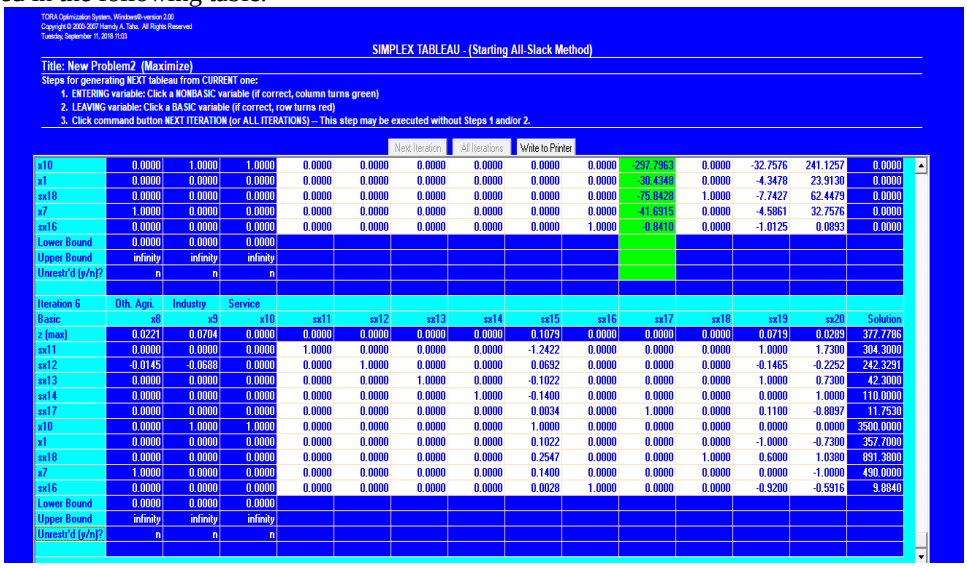
[]

lb =

0 0 0 0 0 0 0 0 0 0

ub =
[]
Optimal solution found.
x =
1.0e ∗ 03∗
0.3577
0
0
0
0
0
0.4900
0
0
3.5000
Z =
-377.7786
ans =
377.7786

In order to get an authentication about the values of the solution, we use the same LPP and solved using TORA. The Tora Output is mentioned in the following table.



TORA Optimization System, Windows® version 2.00
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Tuesday, September 11, 2018 11:03

SIMPLEX TABLEAU - (Starting All-Slack Method)

Title: New Problem2 (Maximize)

Steps for generating NEXT tableau from CURRENT one:
1. ENTERING variable: Click a NONBASIC variable (if correct, column turns green)
2. LEAVING variable: Click a BASIC variable (if correct, row turns red)
3. Click command button NEXT ITERATION (or ALL ITERATIONS) – This step may be executed without Steps 1 and/or 2.

Next Iteration | All Iterations | Write to Printer

x10	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-297.7863	0.0000	-32.7576	241.1257	0.0000
x1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-30.4348	0.0000	-4.3478	23.9130	0.0000
sx18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-75.8478	1.0000	-7.7427	62.4479	0.0000
x7	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-41.6315	0.0000	-4.5861	32.7576	0.0000
sx16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	-0.8410	0.0000	-1.0125	0.0893	0.0000
Lower Bound	0.0000	0.0000	0.0000											
Upper Bound	infinity	infinity	infinity											
Unrestr'd (y/n)?	n	n	n											

Iteration 6	Oth. Agri.	Industry	Service											
Basic	x8	x9	x10	sx11	sx12	sx13	sx14	sx15	sx16	sx17	sx18	sx19	sx20	Solution
z (max)	0.0221	0.0704	0.0000	0.0000	0.0000	0.0000	0.0000	0.1079	0.0000	0.0000	0.0000	0.0719	0.0289	377.7786
sx11	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	-1.2422	0.0000	0.0000	0.0000	1.0000	1.7300	304.3000
sx12	-0.0145	-0.0688	0.0000	0.0000	1.0000	0.0000	0.0000	0.0692	0.0000	0.0000	0.0000	-0.1465	-0.2252	242.3291
sx13	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	-0.1022	0.0000	0.0000	0.0000	1.0000	0.7300	42.3000
sx14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	-0.1400	0.0000	0.0000	0.0000	0.0000	1.0000	110.0000
sx17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0034	0.0000	1.0000	0.0000	0.1100	-0.8097	11.7530
x10	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3500.0000
x1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1022	0.0000	0.0000	0.0000	-1.0000	-0.7300	357.7000
sx18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2547	0.0000	0.0000	1.0000	0.6000	1.0380	891.3800
x7	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1400	0.0000	0.0000	0.0000	0.0000	-1.0000	490.0000
sx16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0028	1.0000	0.0000	0.0000	-0.9200	-0.5916	9.8840
Lower Bound	0.0000	0.0000	0.0000											
Upper Bound	infinity	infinity	infinity											
Unrestr'd (y/n)?	n	n	n											

Output using TORA Version 2.00

This is the last 6th iteration of TORA

4. BANK LOAN POLICY DECISION BY USING BINARY INTE GER PROGRAM

A Bank is devising a loan policy by observing the NPA of five quarter of a financial year. The aim of the bank managerial is to decide which category of loan should be preferred in the coming quarter to reduce the NPA. The numerical data are provided in the following table. Based on this we determine a LPP and solve it using Binary integer program to decide which category loans to be provided in the next quarter in a preference.

Loan/Months	Mar-17	June-17	Dec-17	Mar-18	June-18	Return
Retail	3.12	2.65	2.33	2.11	2.65	48638.26415094340
Agriculture	5.10	7.98	6.06	6.05	6.09	42097.39130434780
Micro & Small	8.92	9.56	11.02	11.99	12.03	42398.17622610140
Medium & Large	15.21	17.36	18.15	23.34	23.25	134456.09677419300

NPA (Non Performing Assists) are unrecoverable and produce no interest revenue.
Strategies to distribution of loan by bank:

1. Total NPA of month Mar-17 should not exceed 11.17.
2. Total NPA of month June-17 should not exceed 12.63.
3. Total NPA of month Dec-17 should not exceed 13.03.

4. Total NPA of month Mar-18 should not exceed 15.73.
5. Total NPA of month June-18 should not exceed 16.00.

Mathematical Models for variable and constraint determination:

Let us assign the variable for loan quantities in crores,

x_1 = Retail,

x_2 = Agriculture,

x_3 = Micro & Small,

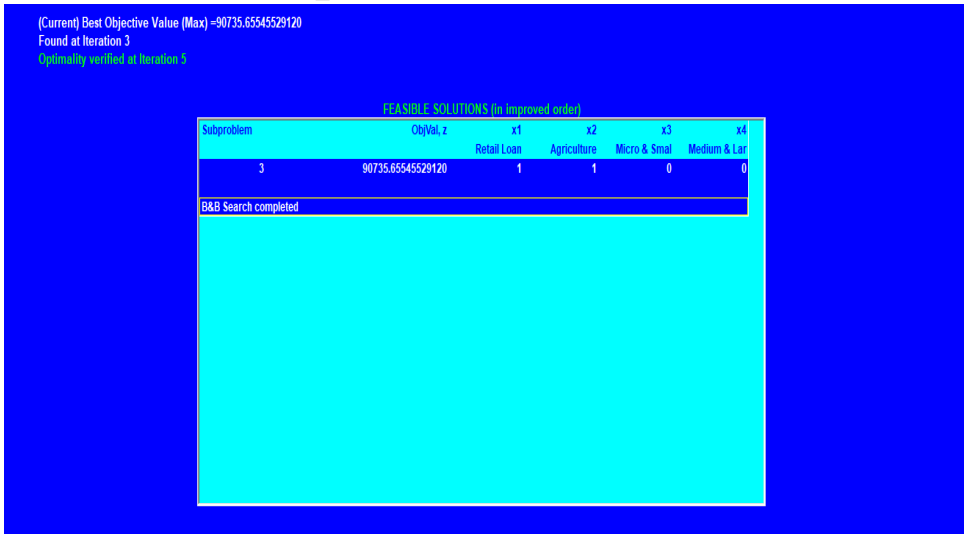
x_4 = Medium & Large.

The objective of the bank managerial is to maximize Return.

Maximize Z = 48638.26415094340x1 + 42097.39130434780x2 + 42398.17622610140x3 + 134456.09677419300x4

The constraints are determined from the investment strategies mentioned in the problem description.

1. Total NPA of month Mar-17 should not exceed 11.17.
   $3.12x1 + 5.10x2 + 8.92x3 + 15.21x4 \leq 11.17$
2. Total NPA of month June-17 should not exceed 12.63.
   $2.65x1 + 7.98x2 + 9.56x3 + 17.36x4 \leq 12.63$
3. Total NPA of month Dec-17 should not exceed 13.03.
   $2.33x1 + 6.06x2 + 11.02x3 + 18.15x4 \leq 13.03$
4. Total NPA of month Mar-18 should not exceed 15.73.
   $2.11x1 + 6.05x2 + 11.99x3 + 23.34x4 \leq 15.73$
5. Total NPA of month June-18 should not exceed 16.00.
   $2.65x1 + 6.90x2 + 12.03x3 + 23.25x4 \leq 16.00$



5. CONCLUSION

The Numerical data used in the formulation of LPP problems mentioned in Section 3 and Section 4 are taken from the web site of Financial year 2017-2018, Union Bank of India [6, 7]. In the Section 3, the LPP solution is x1 = 357.7, x7 = 490.0, x10 = 3500.0 and x2, x3, x4, x5, x6, x8, x9 = 0. The Objective value is z = 377.7786 crores. In the Section 4 binary integer LPP solution, more emphasize the bank investment category of the Retail and Agriculture and lesser emphasize Micro-Small & Medium-Large scale investment.

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